


S matrice simmetrica $V = \mathbb{R}^n$

$$g_S(x, y) = {}^t x \cdot S \cdot y$$

$$x, y \in \mathbb{R}^n \quad x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} \quad y = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

$$\begin{matrix} t_x & S & y & = & \square \end{matrix}$$

Es 7.1.10 nel libro c'è un errore:

$$S = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$$

$$g_S(x, y) =$$

In generale

$$g_S(x, y) = \sum_{i,j=1}^n x_i S_{ij} y_j$$

$$= 2x_1y_1 + \underline{x_1y_2} + \underline{x_2y_1} + \textcircled{x_2y_2}$$

forma quadratic: $q_S(x) = g_S(x, x)$

$$= 2x_1^2 + 2x_1x_2 + x_2^2$$

v e w sono **ORTOGONALI**,

$$g(v, w) = 0$$

• Radicale di g : $V^\perp = \{v \in V : g(v, w) = 0 \ \forall w\} \subseteq V$

• Vettori isotropi: $\{v \in V : g(v, v) = 0\}$

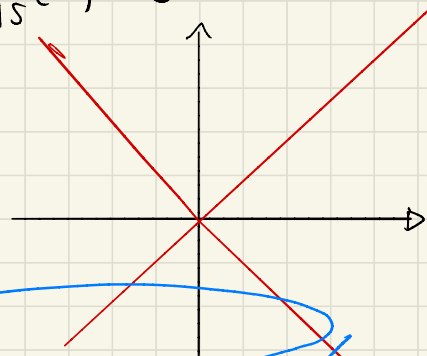
I vettori in $V^\perp$ sono isotropi, ma non sempre vale il viceversa

Es: $S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ←

$x \in \mathbb{R}^2$ isotopo se $g_S(x, x) = 0$ cioè $q_S(x) = 0$

$$q_S(x) = x_1^2 - x_2^2 = 0$$

$$(x_1 + x_2)(x_1 - x_2)$$

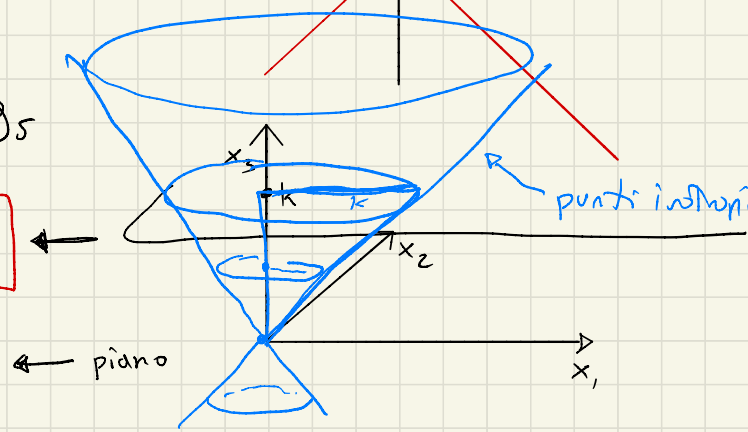


Altro esempi:

$S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ ←

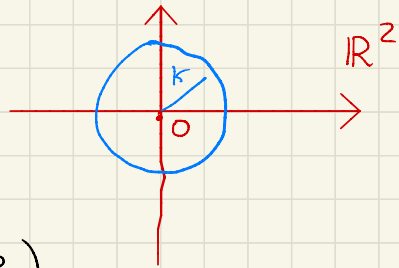
$x \in \mathbb{R}^3$ isotopi per g_S

$$0 = g_S(x, x) = q(x) = \begin{cases} x_1^2 + x_2^2 - x_3^2 = 0 \\ x_3 = k \end{cases}$$



$$x_1^2 + x_2^2 - k^2 = 0$$

$$\underline{x_1^2 + x_2^2 = k^2}$$



Es: $V = M(n, \mathbb{R})$

Date $A, B \in V$ definiremo $g(A, B) = \underline{\text{tr}({}^t A \cdot B)}$

Mostra che è un prodotto scalare su V def +

dim: 1) $g(A + A', B) \stackrel{?}{=} g(A, B) + g(A', B)$

$\text{tr}({}^t A \cdot B) + \text{tr}({}^t A' \cdot B)$

$\text{tr}({}^t(A + A') \cdot B)$

$\parallel$

$$\text{tr}(({}^t A + {}^t A') \cdot B) = \text{tr}({}^t A \cdot B + {}^t A' \cdot B) = \text{tr}({}^t A \cdot B) + \text{tr}({}^t A' \cdot B)$$

OK.

2) esercizio $g(\lambda A, B) = \lambda g(A, B)$

3, 4)

$$5) \quad g(A, B) \stackrel{?}{=} g(B, A)$$

$$\text{tr}(\text{}^t A \cdot B)$$

$$\text{tr}(\text{}^t B \cdot A)$$

$$\text{tr}(\text{}^t(\text{}^t A \cdot B)) = \text{tr}(\text{}^t B \cdot \text{}^t(\text{}^t A))$$

$$\text{}^t(CD) = \text{}^t D \text{}^t C$$

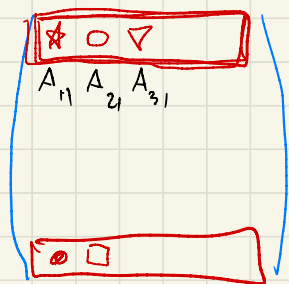
$$(CD)^{-1} = D^{-1} C^{-1}$$

$\bar{E}$ definitivamente positivo: $g(A, A) > 0 \quad \forall A \neq 0$ (TES1)

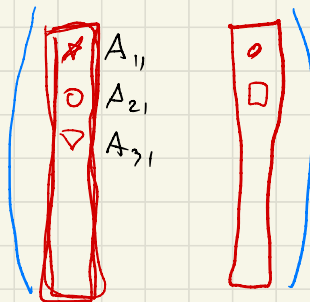
$$g(A, A) = \text{tr}(\text{}^t A \cdot A) = \bigotimes$$

$$A_{ij}$$

$$A_{ji}$$



$\text{}^t A$



A

$$\frac{A_{11}^2 + A_{21}^2 + \dots + A_{n1}^2}{A_{12}^2 + \dots + A_{1n}^2} = \frac{A_{11}^2 + \dots + A_{1n}^2}{A_{1n}^2 + \dots + A_{nn}^2}$$

$$\otimes \quad \underbrace{A_{11}^2 + A_{12}^2 + \dots + A_{nn}^2}_{n^2 \text{ termini}} > 0 \quad \text{se } A \neq 0$$

Es: $V = \mathbb{R}_2[x] = \{\text{polinomi di grado } \leq 2\} = \{a_0 + a_1x + a_2x^2\}$

$$g(p, q) = \underline{p(0)q(0) + p(1)q(1) + p(2)q(2)}$$

è un prodotto scalare (soddisfa i 5 assiomi)

È definito positivo?

dim: $g(p, p) > 0 \quad \forall p \neq 0 \quad \underline{\text{TESI}}$

$$g(p, p) = p(0)^2 + p(1)^2 + p(2)^2 > 0 \quad \forall p \in \underline{\underline{\mathbb{R}_2[x]}}$$

Infatti se $p(0) = p(1) = p(2) = 0$

allora p è il polinomio nullo

perché altrimenti non può avere 3 radici

Es: Stesso g su $V = \mathbb{R}_3[x]$. Mostra che g è DEGENERE

Ricorda che:

$$g \text{ def+} \Rightarrow g(v,v) > 0 \quad \forall v \neq 0$$

$$\Rightarrow g(v,v) \neq 0 \quad \forall v \neq 0$$

$\Rightarrow$ non ci sono vettori isotropi $v \neq 0$

$$\Rightarrow V^\perp = \{0\}$$

$\nRightarrow$ NON È VERA

$\nRightarrow$ NON È VERA

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ non è degenera
ma ha
vettori isotropi

Torniamo a g

Degenera = $\exists$ $p(x)$ ortogonale a tutti, cioè t.c. $g(\underline{p}, q) = 0 \quad \forall q$

$$g(p, q) = \underbrace{p(0)}_{=0} \cdot q(0) + \underbrace{p(1)}_{=0} \cdot q(1) + \underbrace{p(2)}_{=0} \cdot q(2) = 0$$

$$p(x) = x(x-1)(x-2) \quad p(0)=0 \quad p(1)=0 \quad p(2)=0$$

$$g(p, q) = 0 \quad \forall q \in V, \quad (\Rightarrow g(p, p) = 0)$$

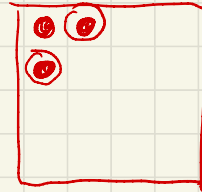
MATRICE ASSOCIATA

g prodotto scalare su V (su $\mathbb{R}$) $\mathcal{B} = \{v_1, \dots, v_k\}$ base di V

Def: La **MATRICE ASSOCIATA** a g nella base $\mathcal{B}$ è la matrice $k \times k$

S dove

$$S_{ij} = g(v_i, v_j)$$



Notiamo che S è simmetrica, cioè

$$S_{ij} = S_{ji} \quad \text{infatti} \quad g(v_i, v_j) = g(v_j, v_i)$$

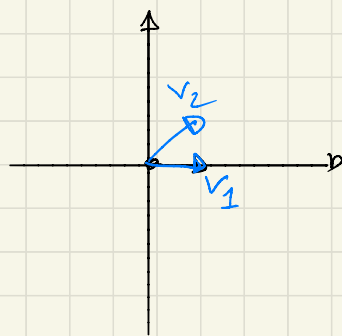
Es: g = Prodotto scalare euclideo su $\mathbb{R}^2$, $\mathcal{B} = \left\{ v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

Allora $S = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$

$$S_{11} = g(v_1, v_1) = (1 \ 0) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$$

$$S_{12} = g(v_1, v_2) = (1 \ 0) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$$

$$S_{22} = g(v_2, v_2) = (1 \ 1) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 2$$



In generale scriviamo

$$S = [g]_{\mathcal{B}}$$

Oss:

$$[g_S]_{\mathcal{C}} = S$$



$$g_S : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$$

$$g_S(x, y) = {}^t x \cdot S \cdot y$$

$$\text{Euclideo: } S = I \quad g_I$$

$$\mathcal{C} = \{e_1, e_2, \dots, e_n\}$$

(simile a fatto sugli endomorfismi)

$$[L_A]_{\mathcal{C}}^{\mathcal{C}} = A$$

Infatti $S = S' = [g_S]_{\mathcal{C}}$

$$s'_{ij} = g_S(e_i, e_j) = \underline{{}^t e_i} S \underline{e_j} = \underline{s_{ij}}$$

"Si può lavorare con la matrice associata:"

$$V, g \quad B = \{v_1, \dots, v_n\} \quad S = [g]_B$$

Vale

$$\rightarrow g(v, w) = {}^t [v]_B \cdot S \cdot [w]_B$$

$$\textcircled{T}: V \rightarrow W$$

$$\textcircled{A} = [T]_B^B$$

Esempio: g Euclideo su $\mathbb{R}^2$

$$g = g_I \quad [g]_e = I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B = \left\{ \underline{v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}}, \underline{v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}} \right\} \quad \text{ALTRA BASE}$$

$$S = [g]_B = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$$

$$\underline{v = \begin{pmatrix} 2 \\ 1 \end{pmatrix}}$$

$$\underline{w = \begin{pmatrix} -1 \\ 1 \end{pmatrix}}$$

$$v \in V \quad T(v) \in W$$

$$[T(v)]_e = \textcircled{A} \cdot [v]_B$$

$A \cdot v$ non ha
senso

$$g(v, w) = {}^t v \cdot w = (2 \ 1) \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -2 + 1 = -1$$

$$g(v, w) = {}^t [v]_{\mathcal{B}} \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} [w]_{\mathcal{B}}$$

$$= (1 \ 1) \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

$$= (1 \ 1) \begin{pmatrix} -1 \\ 0 \end{pmatrix} = -1$$

$$v = \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \underline{1} \cdot v_1 + \underline{1} \cdot v_2 \Rightarrow [v]_{\mathcal{B}} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$w = \begin{pmatrix} -1 \\ 1 \end{pmatrix} = -2v_1 + v_2 \Rightarrow [w]_{\mathcal{B}} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}$$

In generale, negli endomorfismi se cambio base la matrice associata cambia per SIMILITUDINE:

$$A = [T]_{\mathcal{B}}$$

$$B = [T]_{\mathcal{C}}$$

$$A = M^{-1} B M$$

$$[T]_{\mathcal{B}} = [id]_{\mathcal{B}}^{\mathcal{C}} [T]_{\mathcal{C}}^{\mathcal{C}} [id]_{\mathcal{C}}^{\mathcal{B}}$$

$\leftarrow \quad \leftarrow \quad \leftarrow$

$$M = [id]_{\mathcal{C}}^{\mathcal{B}}$$

Per i prodotti scalari, la matrice cambia per **CONGRUENZA**

$$S = [g]_{\mathcal{B}}$$

$$S' = [g]_{\mathcal{B}'}$$

$$(CD)^{-1} = D^{-1}C^{-1}$$

$$S = {}^t M S' M$$

$$M = [id]_{\mathcal{B}'}^{\mathcal{B}}$$

$${}^t(CD) = {}^t D {}^t C$$

Infatti:

$$g(v, w) = {}^t [v]_{\mathcal{B}'} \cdot S' \cdot [w]_{\mathcal{B}'} \quad [w]_{\mathcal{B}'} = M \cdot [w]_{\mathcal{B}}$$

$$= {}^t (M \cdot [v]_{\mathcal{B}}) \cdot S' \cdot M \cdot [w]_{\mathcal{B}}$$

$$= {}^t [v]_{\mathcal{B}} \cdot \underbrace{{}^t M \cdot S' \cdot M}_{\substack{S \\ \text{red}}} \cdot [w]_{\mathcal{B}}$$

$$= {}^t [v]_{\mathcal{B}} \cdot S \cdot [w]_{\mathcal{B}}$$

Def: Due matrici S, S' simmetriche sono **CONGRUENTI** se
 $\exists M$ invertibile t.c. $S' = {}^t M S M$

Es: $\bar{E}$ una relazione di equivalenza

Es: $V = \mathbb{R}_{(2)}[x]$ $g(p, q) = \underline{p(0)q(0) + p(1)q(1) + p(2)q(2)}$
Scrivere matrice associata rispetto a $\mathcal{C} = \{1, x, x^2\}$

$$S = \begin{pmatrix} 3 & 3 & g(1, x^2) \\ 3 & g(x, x) & g(x, x^2) \\ g(1, x^2) & g(x^2, x) & g(x^2, x^2) \end{pmatrix} \quad S_{ij} = g(v_i, v_j)$$

Esempio: $g(1, 1) = 1 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 = 3$

$$g(1, x) = 1 \cdot 0 + 1 \cdot 1 + 1 \cdot 2 = 3$$

Es: Determina il radicale di $g(p, q) = p(0)q(0) + p(1)q(1) + p(2)q(2)$
su $V = \mathbb{R}_{(3)}[x]$

- 1) Scrivere matrice associata $S = [g]_e$ $e = \{1, x, x^2, x^3\}$
2) Determina $\ker S$

$$S = \begin{pmatrix} 3 & 3 & 5 & 9 \\ 3 & 5 & 9 & 17 \\ 5 & 9 & 17 & 33 \\ 9 & 17 & 33 & 65 \end{pmatrix}$$

$$g(p, q) = p(0)q(0) + p(1)q(1) + p(2)q(2)$$

$$g(1, x^2) = 1 \cdot 0 + 1 \cdot 1 + 1 \cdot 4 = 5$$

$$g(1, x^3) = 1 \cdot 0 + 1 \cdot 1 + 1 \cdot 8 = 9$$

$$g(x, x) = 0 \cdot 0 + 1 \cdot 1 + 2 \cdot 2 = 5$$

$$g(x, x^2) = 0 \cdot 0 + 1 \cdot 1 + 2 \cdot 4 = 9$$

$$g(x, x^3) = 0 \cdot 0 + 1 \cdot 1 + 2 \cdot 8 = 17$$

$$g(x^2, x^2) = 0 \cdot 0 + 1 \cdot 1 + 4 \cdot 4 = 17$$

$$g(x^2, x^3) = 0 \cdot 0 + 1 \cdot 1 + 4 \cdot 8 = 33$$

$$g(x^3, x^3) = 0 \cdot 0 + 1 \cdot 1 + 8 \cdot 8 = 65$$

Determina $\text{Ker } S = \text{Ker} \begin{pmatrix} 3 & 3 & 5 & g \\ 3 & 5 & 9 & 17 \\ 5 & 9 & 17 & 33 \\ 9 & 17 & 33 & 65 \end{pmatrix}$ 

So già che $p(x) = x(x-1)(x-2)$ è nel radicale

$$g(p, q) = 0 \cdot q(0) + 0 \cdot q(1) + 0 \cdot q(2) = 0$$

$$[p(x)]_{\mathcal{C}} = [x^3 - 3x^2 + 2x]_{\mathcal{C}} = \begin{pmatrix} 0 \\ 2 \\ -3 \\ 1 \end{pmatrix}$$

deve tornare

$$S \cdot \begin{pmatrix} 0 \\ 2 \\ -3 \\ 1 \end{pmatrix} = 0$$

$$\text{rk } S \leq 3$$

$$\dim \ker S \geq 1$$

$$\begin{pmatrix} 3 & 3 & 5 \\ 3 & 5 & 9 \\ 5 & 9 & 17 \end{pmatrix} \xrightarrow{\text{II} \rightarrow \text{II} - \text{I}} \begin{pmatrix} 3 & 3 & 5 \\ 0 & 2 & 4 \\ -1 & 3 & 7 \end{pmatrix} \xrightarrow{\text{I} \rightarrow \text{I} + 3\text{III}} \begin{pmatrix} 0 & 12 & 26 \\ 0 & 2 & 4 \\ -1 & 3 & 7 \end{pmatrix}$$

$$\det \neq 0 \Rightarrow \text{rk } S = 3$$

$$\Rightarrow \dim \ker S = 1$$

$$\Rightarrow \ker S = \text{Span} \left(\begin{pmatrix} 0 \\ 2 \\ -3 \\ 1 \end{pmatrix} \right)$$

qui siamo in coordinate, per rispondere dobbiamo tornare al linguaggio di V

$$V^\perp = \ker S \quad \text{NON PROPRIO!}$$

RADICALE
POLINOMI

VETTORI
COLONNA

$$\ker S \subseteq \mathbb{R}^4$$

$$V^\perp \subseteq V$$

$$\Rightarrow V^\perp = \text{Span} (2x - 3x^2 + x^3) = \text{Span} (\underline{x(x-1)(x-2)}) //$$

$$V^\perp = \left\{ \lambda x(x-1)(x-2) \mid \lambda \in \mathbb{R} \right\}$$

SOTTOSPAZIO ORTOGONALE

V sp. vettoriale su $\mathbb{R}$, g prodotto reale su V

$W \subseteq V$ sottospazio

Def: Il SOTTOSPAZIO ORTOGONALE a W è l'insieme

$$W^\perp = \left\{ \underline{v \in V} \mid \underline{g(v, w) = 0} \quad \forall w \in \underline{W} \right\}$$

duale $\underline{V}^\perp = \left\{ v \in V \mid g(v, w) = 0 \quad \forall w \in \underline{V} \right\}$

Prop: $W^\perp$ è un sottospazio vettoriale

dim: già visto per $V^\perp$

1) $0 \in W^\perp$

2) $v, v' \in W^\perp \Rightarrow v + v' \in W^\perp$

3) $v \in W^\perp$
 $\lambda \in \mathbb{R}$
 $\Rightarrow \lambda v \in W^\perp$

$$1) \underline{0 \in W^\perp} \iff \underline{g(0, w) = 0 \quad \forall w \in W} \quad g(0, w) = 0 \text{ sempre}$$

$$2) \underline{v, v' \in W^\perp} \stackrel{?}{=} v + v' \in W^\perp$$

$$\underline{\text{Tesi:}} \quad g(v + v', w) = 0 \quad \forall w \in W \quad \text{OK}$$

$$\begin{array}{ccc} g(v, w) + g(v', w) & \text{perché } w \in W \\ \parallel & \parallel \\ 0 & 0 & v, v' \in W^\perp \end{array}$$

3) esercizio.

□

Prop: Se $W = \text{Span}(w_1, \dots, w_k)$ allora

$$W^\perp = \{v \in V \mid g(v, w_1) = 0, \dots, g(v, w_k) = 0\}$$

Cioè $v \in W^\perp$ se e solo se è ortogonale a $w_1, \dots, w_k$ cioè $g(v, w_i) = 0 \quad \forall i = 1, \dots, k$

dim: Se v è ortogonale a $w_1, \dots, w_k$, allora è
 ortogonale a qualsiasi $w \in W$. Infatti:

$$w = \alpha_1 w_1 + \dots + \alpha_k w_k$$

$$\begin{aligned} \underset{=0}{g(v, w)} &= g(v, \alpha_1 w_1 + \dots + \alpha_k w_k) \\ &= g(v, \alpha_1 w_1) + \dots + g(v, \alpha_k w_k) \\ &= \alpha_1 \underset{=0}{g(v, w_1)} + \dots + \alpha_k \underset{=0}{g(v, w_k)} = \alpha_1 \cdot 0 + \dots + \alpha_k \cdot 0 = 0 \end{aligned}$$

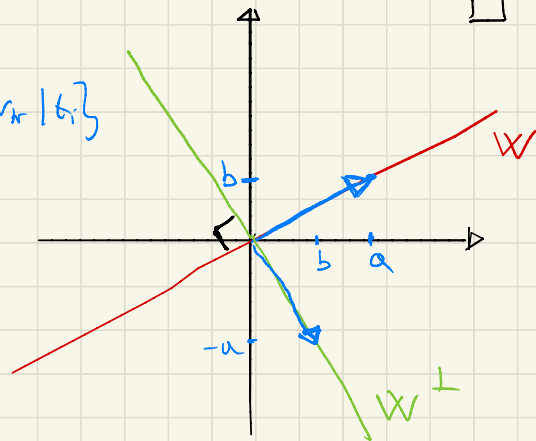
Esempi: $\mathbb{R}^2$, g Euclideo

$$W = \text{Span} \left(\begin{pmatrix} a \\ b \end{pmatrix} \right)$$

PARAMETRICA = $\text{Span}(v_1, \dots, v_k)$
 ESPlicita

$$W^\perp = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \mid g\left(\begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix}\right) = 0 \right\}$$

$$\{t_1 v_1 + \dots + t_k v_k \mid t_i \in \mathbb{R}\}$$



$$= \left\{ (x \ y) \begin{pmatrix} a \\ b \end{pmatrix} = 0 \right\} = \{ \textcircled{a}x + \textcircled{b}y = 0 \}$$

CARTESIANA

"
soluzione di equazioni:

IMPLICITA

$$= \text{Span} \begin{pmatrix} b \\ -a \end{pmatrix}$$

$$a \cdot b + b \cdot (-a) = 0$$

$$g\left(\begin{pmatrix} a \\ b \end{pmatrix}, \begin{pmatrix} b \\ -a \end{pmatrix}\right) = ab - ba = 0$$

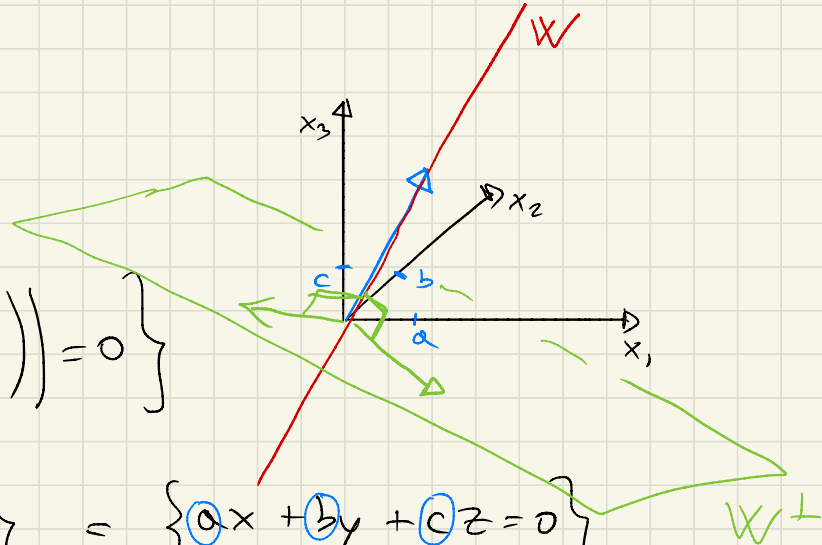
In $\mathbb{R}^3$, g euclideo

$$W = \text{Span} \begin{pmatrix} \textcircled{a} \\ \textcircled{b} \\ \textcircled{c} \end{pmatrix} \quad \text{PARAM.}$$

$$W^\perp = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid g\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix}\right) = 0 \right\}$$

$$= \left\{ (x \ y \ z) \cdot I \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \right\} = \{ \textcircled{a}x + \textcircled{b}y + \textcircled{c}z = 0 \}$$

CART.



CART. $\Rightarrow$ PARAM.

$$\{ax + by + cz = 0\} = \text{Span} \left(\begin{pmatrix} b \\ -a \\ 0 \end{pmatrix}, \begin{pmatrix} c \\ 0 \\ -a \end{pmatrix} \right)$$

can $a, b, c \neq 0$

Sempre in $\mathbb{R}^3$

$$W = \text{Span} \left(\begin{pmatrix} a \\ b \\ c \end{pmatrix}, \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right) \quad \text{Chi è } W^\perp?$$

independent

$$W^\perp = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \mid \underbrace{g \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} a \\ b \\ c \end{pmatrix} \right) = 0}, \underbrace{g \left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}, \begin{pmatrix} d \\ e \\ f \end{pmatrix} \right) = 0} \right\}$$

$$\textcircled{\star} = \begin{cases} ax + by + cz = 0 \\ dx + ey + fz = 0 \end{cases} \begin{matrix} \pi_1 \\ \pi_2 \end{matrix}$$

independent

retta.

CARTESIANA

/

$$\text{rk} \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix} = 2$$

$$\Rightarrow 3 - 2 = 1$$

$$\dim W^\perp$$

PARAMETRICA

$$W^\perp = \text{Span} \begin{pmatrix} g \\ h \\ i \end{pmatrix}$$

Sistemi $\begin{pmatrix} g \\ h \\ i \end{pmatrix}$ risolvendo il sistema $\star$

Formula:

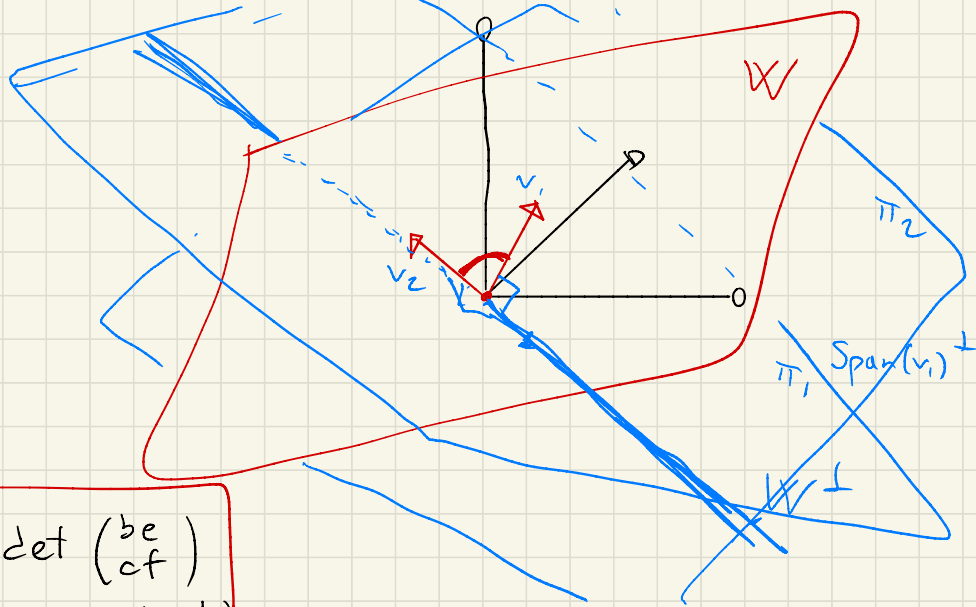
$$g = ae - bd = \det \begin{pmatrix} b & e \\ c & f \end{pmatrix}$$

$$h = cd - af = -\det \begin{pmatrix} a & d \\ c & f \end{pmatrix}$$

$$i = ae - bd = \det \begin{pmatrix} a & d \\ b & e \end{pmatrix}$$

$$\begin{pmatrix} g \\ h \\ i \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \times \begin{pmatrix} d \\ e \\ f \end{pmatrix}$$

prodotto vettoriale
(perché?)



non è importante

Esercizi: 7.1 $\longrightarrow$ 7.7 escluso 7.2 (per mercoledì prossimo)

$$\begin{cases} ga + hb + ic = 0 \\ gd + he + if = 0 \end{cases}$$



$$\begin{cases} ax + by + cz = 0 \\ dx + ey + fz = 0 \end{cases}$$



$\begin{pmatrix} g \\ h \\ i \end{pmatrix}$ è soluzione

6.2 6.3 6.11 6.12 da fare venerdì.